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Effects of Missile and Target Acceleration

Parameters on Time-To-Go Estimation

Final Technical Report

AFOSR-80-0075

By

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### Abstract

Short range air-to-air missiles possess high maneuverability and require real time computation in constructing their commanded controls. The intercept time may be in the order of two to three seconds. Such time constraints necessitate sophisticated control laws, many of which require a good estimate of time-to-go before intercept. Preliminary investigations of a time-to-go estimate using optimal control theory under certain assumptions have yielded promising results.

The estimation algorithm is based upon a performance index consisting of terminal miss-distance and time. Using a state space representation for the missile-target dynamics, guidance law algorithms, and conventional assumptions on system dynamics, the time-to-go estimate has shown some improvement over existing methodologies.

However, several assumptions on system dynamics which have been used in the development of the time-to-go estimate may be unrealistic under current missile technology. The purpose of this research effort was to extend the existing methodologies and eliminate some of the assumptions so mentioned. In particular, the assumption of unlimited acceleration commands may be eliminated from the time-to-go estimation by incorporating acceleration constraints directly as boundary conditions on the optimal control law. To compensate for the zero target acceleration assumption, one may apply a weighting functional of the missile parameters to the estimator performance measure. Such extensions yield promising results.

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Table of Contents

	<u>Pg.</u>
I. Introduction	1
II. Problem Development	10
A. Limited Acceleration Commands	12
B. Modification of the Time-To-Go Estimator	27
III. Dynamic Output Feedback	35
IV. Application of the Error Excitation Approach to the Finite Terminal Time Case	39
V. Summary and Recommendations	41
VI. References	43

## I. Introduction

To investigate short range air-to-air missile scenarios, several issues need be addressed: (1) modelling the missile and target dynamics; (2) simulating the target maneuver strategy; (3) developing a control law for the missile in order to intercept the target and (4) generating the information necessary to apply this control law. Points (3) and (4) are usually handled conjunctively and the evaluation of the control designs are done under real time constraints.

Modelling the missile and target dynamics can be handled in a linear or nonlinear, time-invariant or time-varying fashion. Nonlinearities appear from saturation coefficients and current missile capabilities. They may also appear from the choice of coordinate systems (inertial, missile body, seeker coordinates, etc.) and states. This research effort assumes a linearized, time-invariant model.

Development of a missile control law for target intercept is based upon the chosen missile-target model. Work has been done [15] in applying various fields of optimal and stochastic control to the air-to-air intercept problem which has shown some promise. Application of optimal control theory results in transforming system performance objectives into a mathematical cost functional to be optimized. As terminal miss-distance is of primary importance, this performance objective can be described by:

$$J = \underline{x}^T(t_f) S_f \underline{x}(t_f) \quad (1)$$

where  $\underline{x}(t_f)$  represents aerodynamic states (relative positions, velocities and possibly accelerations) and  $S_f$  is an appropriate weighting matrix on position only.

Equation (1) represents a primary system objective; however, it is usually desirable to hit the target under other constraints as well (minimal time, minimal control effort, minimal fuel consumption, and so on). Anderson [1] has investigated various cost functionals under the assumption that a planar linearized model adequately describes the air-to-air scenario. It was found that a cost functional of the form:

$$J = \underline{x}^T(t_f) S_f \underline{x}(t_f) + b \int_{t_0}^{t_f} \underline{u}^T(t) R \underline{u}(t) dt \quad (2)$$

gave adequate intercept characteristics. The parameter  $b$  represents a weighting on control effort. It was also found that knowledge of present and future accelerations generally improved intercept characteristics. However, these parameters are not generally known.

Preliminary results of the guidance law methodologies discussed above indicate that there are two important parameters that need be estimated or predicted accurately in order to improve system performance: a good estimate of target acceleration,  $\underline{a}^T$ , and a good estimate of time-to-go before intercept,  $t_{g_0}$ .

Early attempts to estimate time-to-go resulted in using range  $R$  and range rate  $\dot{R}$  in proportional navigation guidance. Effectively, the estimate:

$$t_{g_0} = -\frac{\dot{R}}{R} \quad (3)$$

represents the time-to-go estimate of pro-navigation guidance which gave adequate results for simple target maneuvers. However, the method degenerated for scenarios with non-zero off-boresight yaw angles BAY (the angle between missile body acceleration in the  $x$ -direction and line-of-sight vector at launch).

Riggs [14] has developed a time-to-go estimate under the assumption that acceleration along the line-of-sight can be approximated by the measurable achieved missile acceleration in the x-direction. This resulted in the iterative time-to-go estimate of:

$$t_{g_0}^{(i+1)} = \frac{2R}{v_c + \sqrt{v_c^2 + 4 \hat{A} R g}} \quad (4)$$

where:

$$\hat{A} = \frac{101.4 - 39t - 12t_{g_0}^{(i)}}{t_{g_0}^{(i)}}$$

and  $g = 32.174 \text{ ft/sec}^2$ . Using this estimate of time-to-go with a suboptimal control law based upon (2) gave better inner launch boundaries (minimum launching range in which missile still hits target) from that of (3) using either pro-nav or the suboptimal control law. (see Figure 1)

Lee [10] has constructed a secondary cost functional of the form:

$$J = \underline{x}^T(t_f) S_f \underline{x}(t_f) + q(\int dt)^2 \quad (5)$$

in order to estimate time-to-go. Under the assumption of zero target accelerations and essentially unlimited acceleration commands, this resulted in the iterative time-to-go estimate:

$$t_{g_0} = - \frac{\underline{x}_p^T(t) D F \underline{x}_v(t)}{\underline{x}_v^T(t) F^T F \underline{x}_v(t) + q} \quad (6)$$

where  $\underline{x}_p(t)$  represents relative position in missile body coordinates,  $\underline{x}_v(t)$  represents relative velocity in missile body coordinates, and

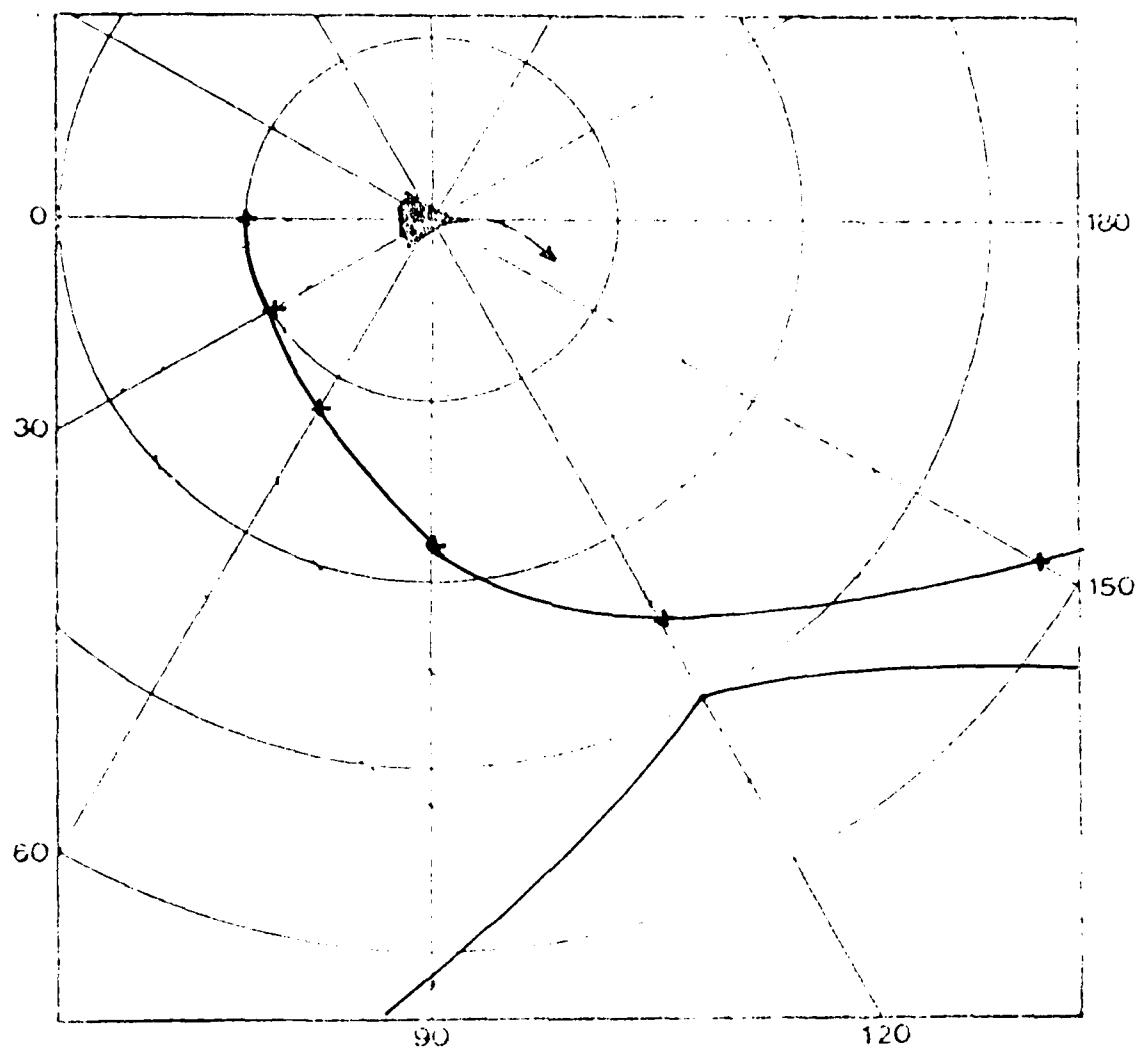


Figure 1: Inner launch boundary for 40° off bore-sight angle.

— : Pro-Nav  
—+— : Guidance law of [14]

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \end{bmatrix}$$

$$d = e^{-3/2} \cos \left( \frac{\sqrt{3}}{2} \right) + \sqrt{3} e^{-3/2} \sin \left( \frac{\sqrt{3}}{2} \right)$$

$$f = \frac{2}{\sqrt{3}} e^{-3/2} \sin \left( \frac{\sqrt{3}}{2} \right). \quad (7)$$

The estimate (6) was based upon a system dynamic and control representation. Preliminary simulation results have shown improvement over proportional navigation and (4) at low and mid aspect angles (angle between the target's velocity vector and the line-of-sight vector at launch). Appealing attributes of using (6) as an estimate for time-to-go include its minimal computation, its iterative nature and the insight one may obtain of the influence of design objectives (minimal terminal miss-distance and overall time) on the estimate of time-to-go. (see Figure 2)

The time-to-go estimator described in (6), however, assumes unlimited acceleration commands and zero target acceleration. These assumptions may be unrealistic in practice. Furthermore, the guidance control laws described in [1] and [14] may require complete accessibility or knowledge of all of the system states.

This investigation addresses the issues of the time-to-go estimation problem under limited acceleration commands. Further, the effects of missile parameters on the time-to-go parameter will be discussed. In particular,

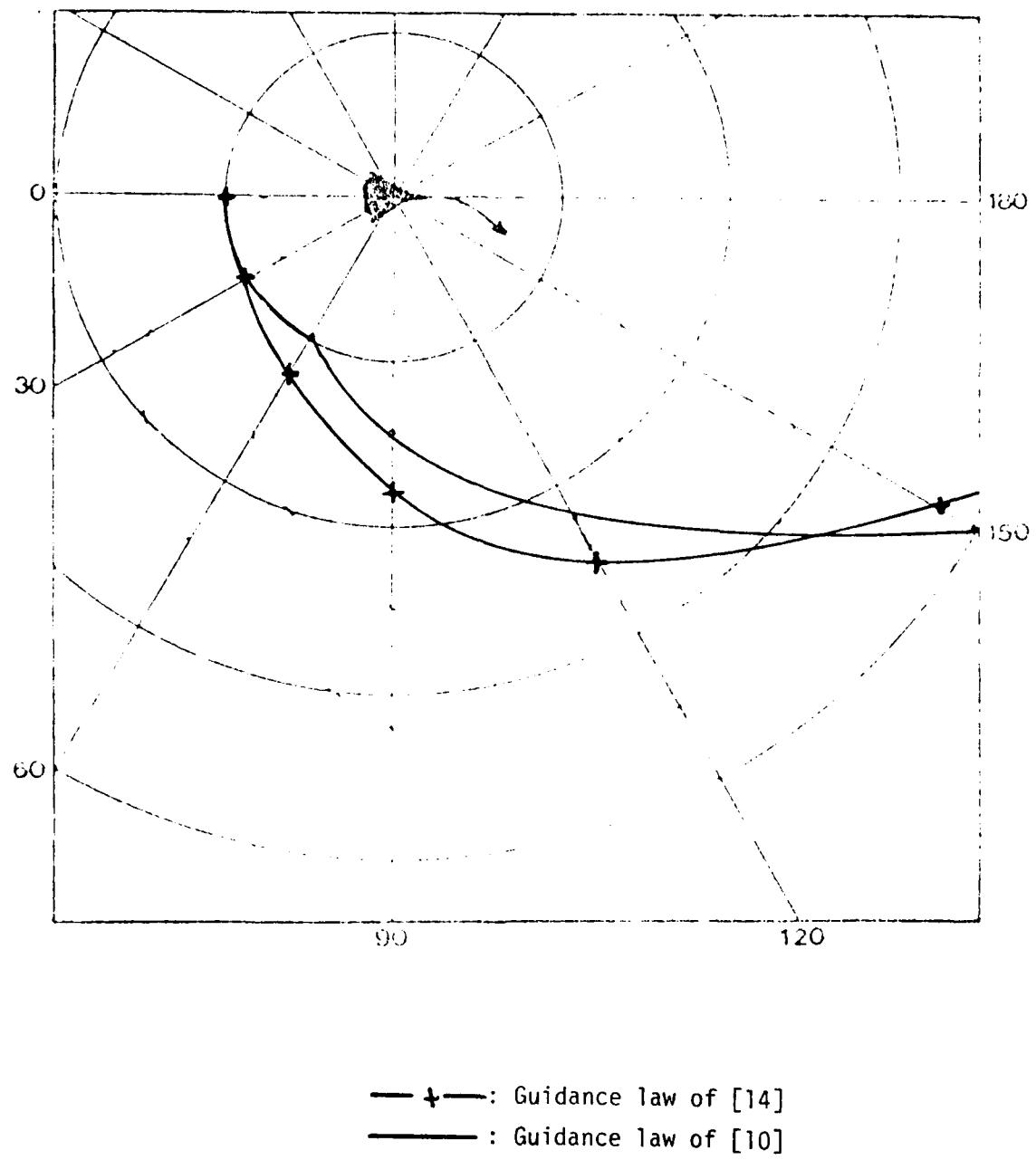


Figure 2a: Inner launch boundary comparison for two guidance control laws;  
Off bore-sight angle is  $40^\circ$ .

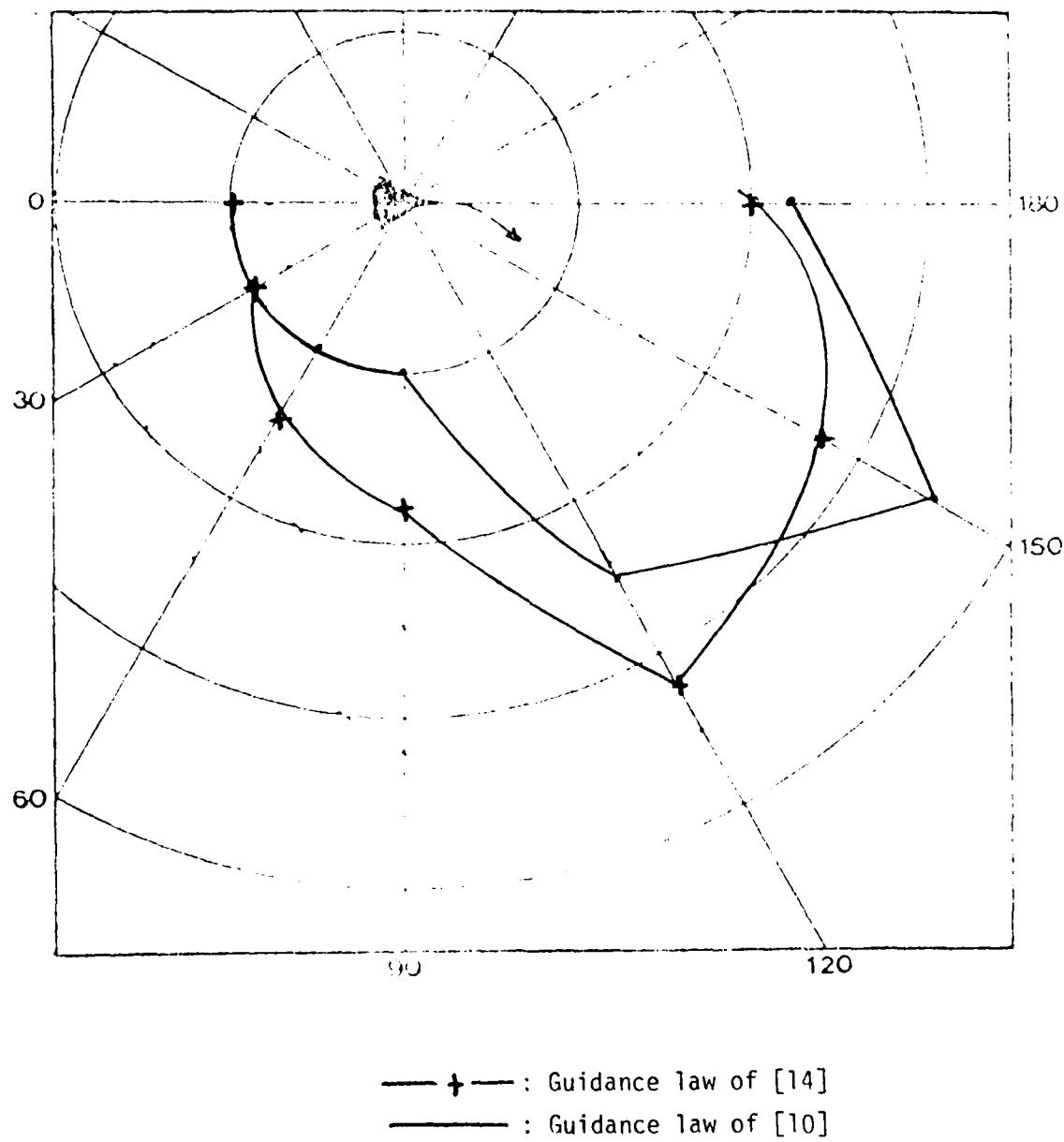


Figure 2b: Inner launch boundary comparision for two guidance laws;  
Off bore-sight angle is  $0^\circ$ .

the influence of a weighting functional of system parameters on the estimator convergence properties will be examined. It will be shown that knowledge of the initial aspect and off bore-sight angles may improve the time-to-go estimator and thereby improve the inner launch boundary of the air-to-air scenario.

While investigating the influence of system parameters of the time-to-go estimate, it was felt that research effort was also required to improve the guidance laws necessary for optimal target intercept. That is, in order to further eliminate the target acceleration assumption from the optimal guidance design, one must also reformulate the guidance laws and modify the linear regulation cost functional to be optimized.

The approach of linear regulation is to drive the system states of outputs to desired trajectories in an optimal fashion. The particular specifications of the system dictate the form of the performance measure.

In many applications, however, due to imperfect or incomplete measurements of the system states as well as the controller structure constraints, only the system's outputs are available for compensation. This is the case in the AAM problem in which the target acceleration is not known or can not be measured, and the controller's parameters are restricted or constrained due to physical reasons.

Methods have been suggested for output feedback regulation in which the control law is constrained to certain structures [8, 9, 18]. These algorithms, however, may lack assurance of system stability, may not possess the flexibility to include dynamic compensation or arbitrary constraints in controller structures, or may not include knowledge of the influence of the compensator parameters and order on the state or output trajectories.

A design algorithm for output feedback has been designed by the author, based upon construction of an error vector between optimal trajectories when state feedback is available and the trajectories when only the outputs are available. Minimization of the performance measure based upon this error vector leads to a Riccati-type matrix equation which may be solved in an iterative fashion. The methodology is iterative in nature for both compensator order as well as compensator parametric search. An appealing feature of this approach is that the state trajectories using state feedback may be directly compared with those obtained using dynamic or structured output feedback compensation. The application of this algorithm to construction of guidance laws is an area of future research.

The results of this research effort will be developed in the following manner: Section II will discuss the optimal guidance law construction and the time-to-go estimation problem from the state space approach. The effects of limited acceleration commands on the time-to-go estimate and miss-distance values will be discussed. Further, the effects of varying a weighting functional in the time-to-go estimator will also be discussed here. The results will illustrate some improvement in the inner launch boundary of the AAM scenario. In Section III the design methodology for dynamic compensation will be introduced. A discussion of the application of this methodology to an improved guidance law is presented in Section IV. This may be an area of further investigation.

### II. Problem Development

Consider the following dynamics representing the air-to-air intercept problem:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \quad (8)$$

where

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

The states  $\underline{x}(t)$  represent relative positions and velocities between missile and target, while  $\underline{u}(t)$  is the commanded control vector.

In order to achieve intercept with minimal control effort, a performance measure may be constructed as:

$$J = \underline{x}^T(t_f) S_f \underline{x}(t_f) + \int_{t_0}^{t_f} \underline{u}^T(t) R \underline{u}(t) dt \quad (9)$$

where

$$S_f = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \quad R = \text{diag}(r), \quad t_0 = \text{initial time},$$

and  $t_f$  final time. Assuming that all the states are accessible, the maximum principle [3] yields the optimal control law:

$$\begin{aligned} \underline{u}^*(t) &= F^*(t) \underline{x}^*(t) \\ F^*(t) &= -R^{-1} B^T P^*(t) \end{aligned} \quad (10)$$

where  $P^*(t)$  satisfies:

$$\dot{P}(t) + A^T P(t) + P(t) A - P(t) B R^{-1} B^T P(t) = 0 \quad (11)$$

and

$$\begin{aligned} \underline{x}^*(t_0) &= \underline{x}_0 \\ P(t_f) &= S_f \end{aligned} \quad (12)$$

Further, the optimal state trajectory is governed by:

$$\dot{x}^*(t) = (A + B F^*(t)) \underline{x}^*(t) \quad (13)$$

Equations (11) and (13) with boundary conditions(12) constitute a two-point boundary value problem. It may be solved by numerical techniques [5].

In the case when  $r = 0$ , optimization of (9) with respect to  $u(t)$  finds:

$$\underline{u}^*(t) = F^* \underline{x}^*(t) \quad (14)$$

where

$$F^* = -\frac{3}{t_{g_0}^2} [I \quad t_{g_0} I] \quad (15)$$

and  $t_{g_0}$  is defined as  $t_f - t$ , that is, the time before intercept.

The optimal controller defined by (14) requires complete knowledge of all the states as well as knowledge of  $t_{g_0}$ . Furthermore, estimation of  $t_{g_0}$  requires assumptions of zero target acceleration and unlimited accelerations as well as complete access of the states, as seen by (6). Note that (8) is uncontrollable and thus (14) represents a suboptimal control law. This law may be improved if an accurate estimate of  $t_{g_0}$  is obtained.

Under system assumptions so mentioned, one may construct a secondary performance measure to estimate  $t_{g_0}$  as in (5). This measure represents terminal miss distance and minimal time. The weight  $q$  is chosen according to the importance one places on each term. In [10], a constant normalizing weight was chosen.

### A. Limited Acceleration Commands

Consider modifying the control law of (14) to include limitations on the control commands. Two methods may be applied to do this.

#### 1. Slack Variable Approach

Suppose the control constraints are defined as  $m_i$  where  $i=1,2,3$  represents the missile's acceleration controls in the X,Y,Z coordinates in the air-to-air missile case. Let

$$-m_i \leq u_i(t) \leq m_i \quad (16)$$

describe the bounds on the control.

Then (9) may be modified to incorporate (16) as:

$$J = \underline{x}^T(t_f)S_f\underline{x}(t_f) + \int_{t_0}^{t_f} \underline{u}^T(t)R\underline{u}(t) + \underline{g}^T(t)\underline{h}(t) dt \quad (17)$$

where

$$h_i(t) = z_i^2(t) - (m_i - u_i(t))(u_i(t) + m_i) \quad (18)$$

and  $z_i(t)$  is a parametric variable. Then minimization of (17) yields the optimal constrained control. Although this method is simple to implement, there may be numerical difficulties when the system's dimension is high or the system is uncontrollable. This is the case here; therefore, a more direct solution is desired.

## 2. Linear Regulation with Bounded Controls

The constrained control problem for linear regulation may be formulated as:

$$\begin{aligned} J &= \frac{1}{2} \underline{x}^T(t_f) S_f \underline{x}(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \underline{x}^T(t) Q \underline{x}(t) + \underline{u}^T(t) R \underline{u}(t) dt \\ \text{subject to: } \dot{\underline{x}} &= A \underline{x} + B \underline{u} \\ |\underline{u}| &\leq M \end{aligned} \quad (19)$$

Application of the maximum principle yields the Hamiltonian:

$$\begin{aligned} H &= \frac{1}{2} (\underline{x}^T(t) Q \underline{x}(t) + \underline{u}^T(t) R \underline{u}(t)) + \underline{g}^T(A \underline{x}(t) + B \underline{u}(t)) \\ &\quad + \underline{h}_1^T(\underline{u}(t) - M) + \underline{h}_2^T(-\underline{u}(t) - M) \end{aligned} \quad (20)$$

Then

$$\frac{\partial H}{\partial \underline{u}} = \underline{u}^T(t) R + \underline{g}^T B + \underline{h}_1^T - \underline{h}_2^T = 0 \quad (21)$$

The optimal control law under control constraints becomes:

$$\underline{u}(t) = \begin{cases} -R^{-1}B^T \underline{g} & |\underline{u}(t)| \leq M \\ -R^{-1}(B^T \underline{g} + \underline{h}_1 - \underline{h}_2) & \text{otherwise} \end{cases} \quad (22)$$

That is:

$$\underline{u}(t) = \begin{cases} -R^{-1}B^T P_x(t) & |\underline{u}(t)| \leq M \\ M & \underline{u}(t) > M \\ -M & \underline{u}(t) < -M \end{cases} \quad (23)$$

Thus, a clipping function may be applied to (14) to yield the optimal constrained control law. To see this effect on the time-to-go estimate,

simulation studies on the nonlinear, time-varying model of the AAM scenario was performed using (14) with no limits on  $u(t)$ . This yielded the acceleration commands shown in Figures 3 and 5 for two cases. The associated time-to-go estimates are shown in Figures 4 and 6.

It is seen that the time-to-go estimate using (6) generally requires smaller acceleration commands than (4) for these cases. To obtain better insight into the acceleration controls, plots of the acceleration along the line-of-sight were made for the case of an off bore-sight angle of  $0^\circ$ , for various aspect angles. These are shown in Figures 7 through 10. From this, it is seen that a slightly more conservative initial trajectory of the line-of-sight acceleration (between the 0 to 0.4 second period, when the missile is clearing its launch platform) leads to a hit.

The scenario corresponding to Figure 3 was performed using an acceleration command constraint of  $75 \text{ ft/sec}^2$  on all of the control coordinates. The following table summarizes the results:

<u>Acceleration commands</u>		
<u>X-Direction</u>	<u>Y-Direction</u>	<u>Z-Direction</u>
22.886	19.204	0.00
19.264	55.862	- .616
25.672	75.00	-2.183
39.017	75.00	-5.017
61.554	75.00	-10.111
75.00	75.00	-65.733
75.00	75.00	75.00
75.00	75.00	-72.935
75.00	75.00	75.00
75.00	75.00	-44.805

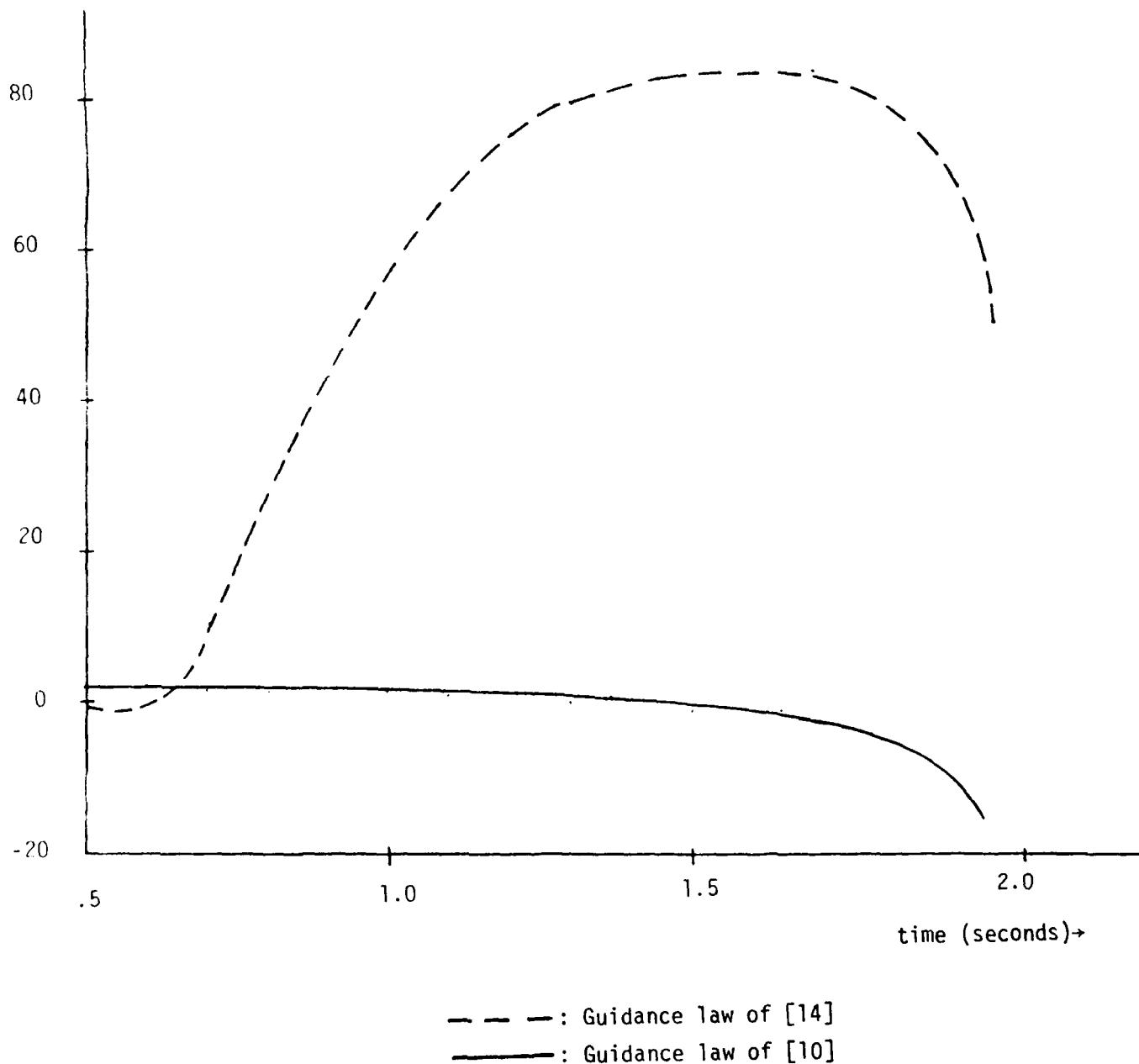


Figure 3a: Acceleration command in the X-direction for  $0^\circ$  aspect angle,  
 $40^\circ$  off bore-sight angle, and 1000' initial range.

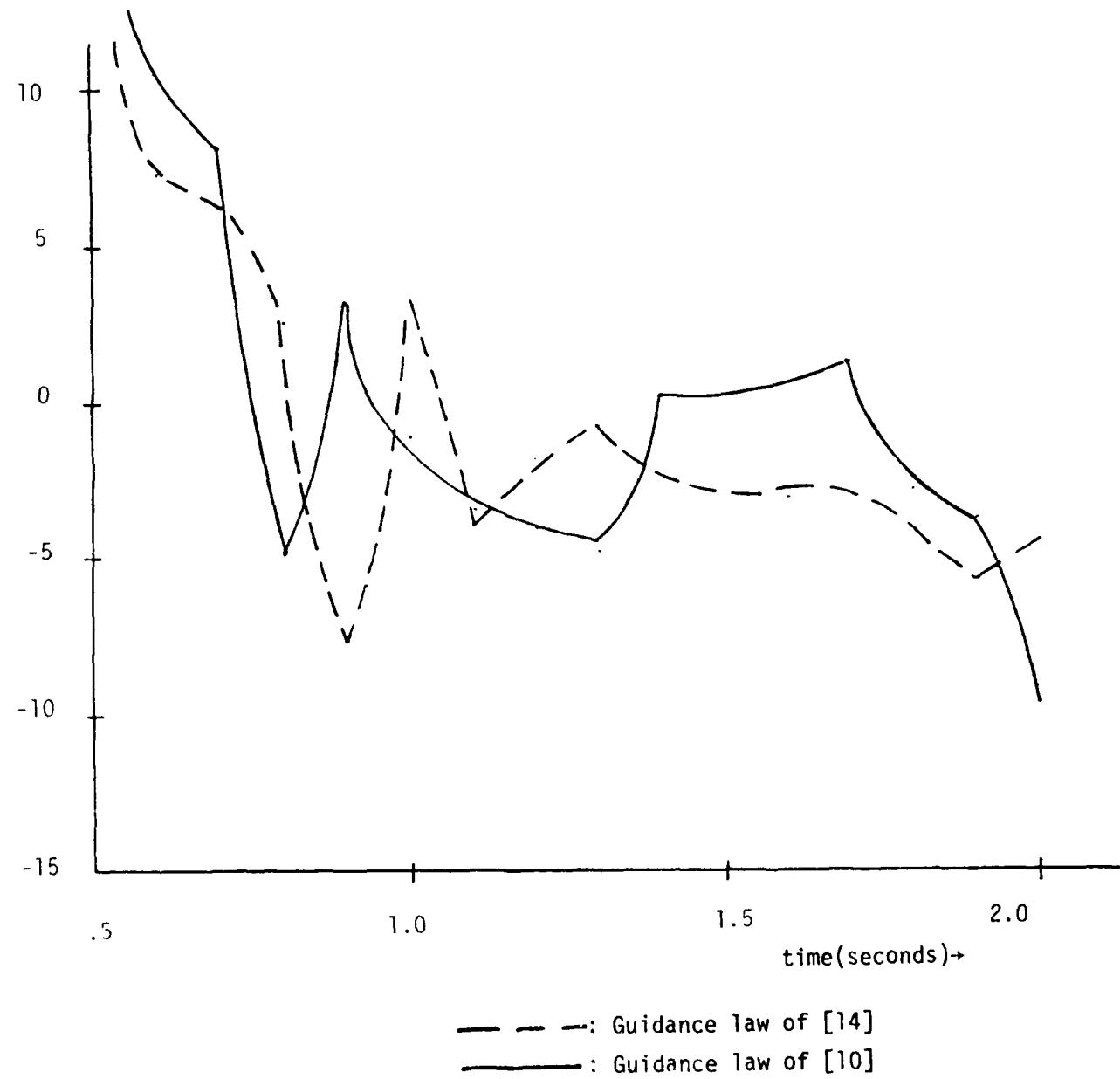


Figure 3b: Acceleration command in the Y-direction for  $0^\circ$  aspect angle,  
 $40^\circ$  off bore-sight angle, and 1000' initial range.

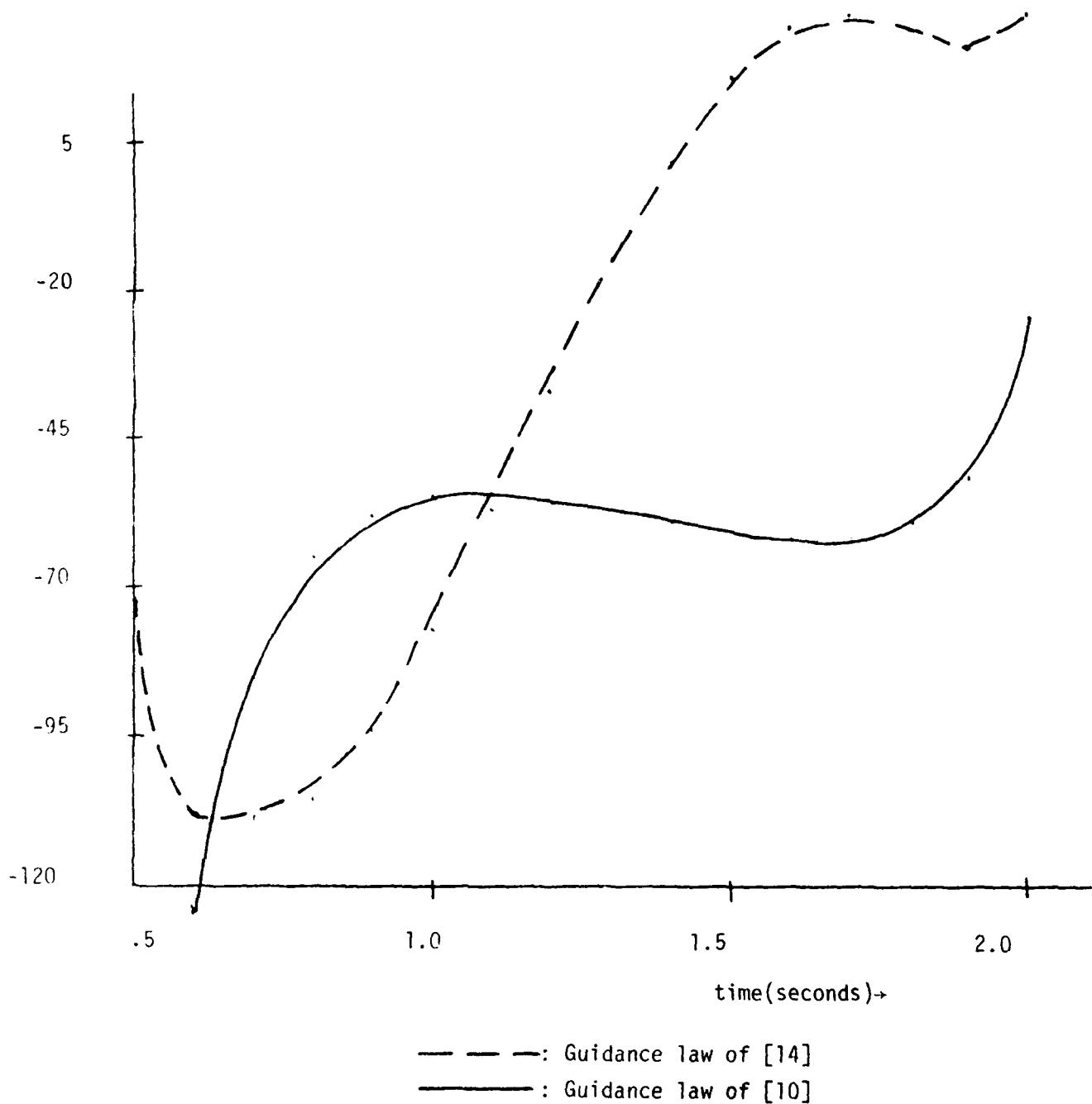


Figure 3c: Acceleration command in the Z-direction for  $0^\circ$  aspect angle,  
 $40^\circ$  off bore-sight angle, and 1000' initial range.

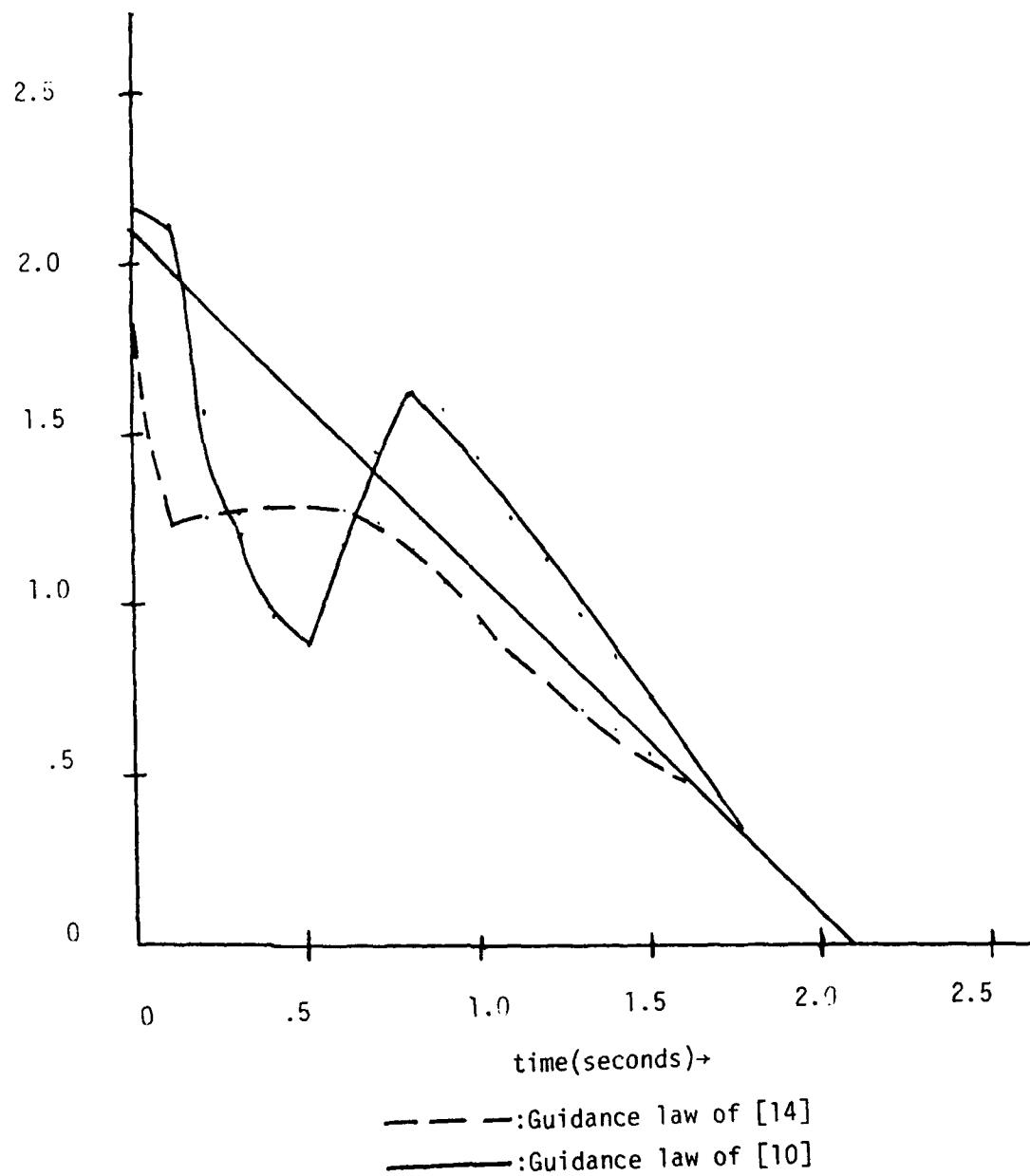


Figure 4: Time-to-go estimate versus true time for  $0^\circ$  aspect angle,  
 $40^\circ$  off bore-sight angle, and 1000' initial range.

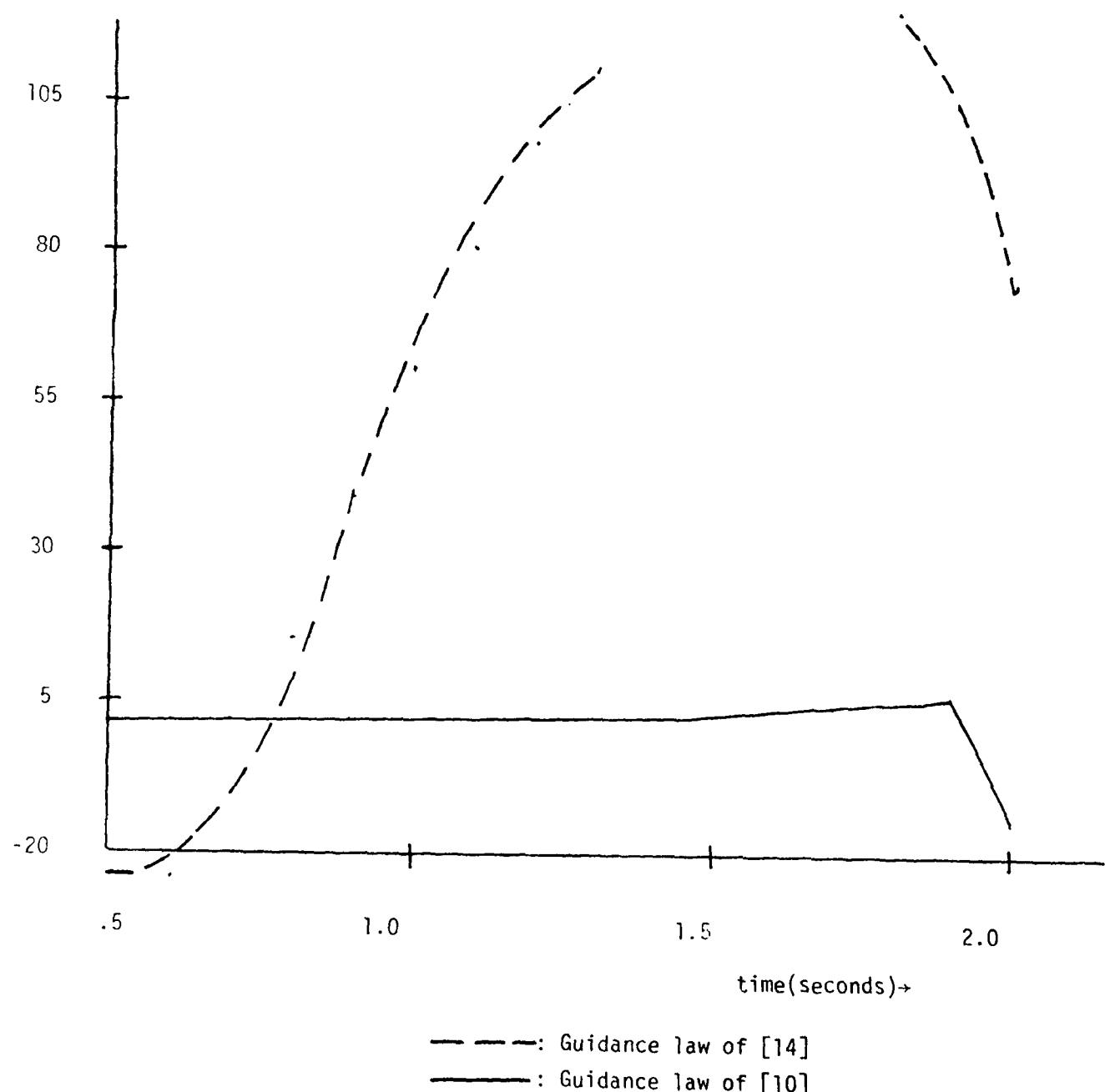


Figure 5a: Acceleration command in the X-direction for  $150^\circ$  aspect angle,  $40^\circ$  off bore-sight angle, and 3750' initial range.

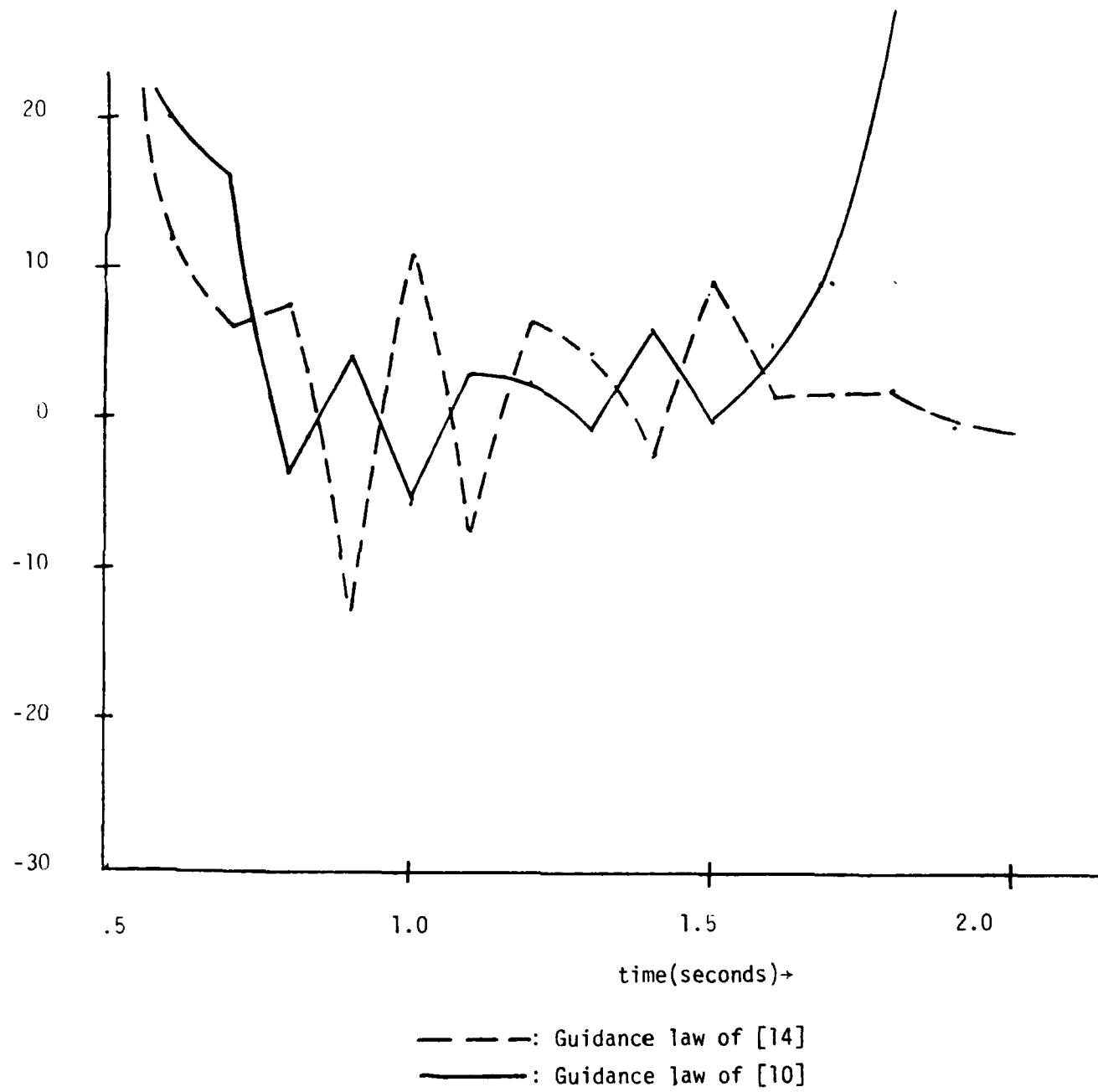


Figure 5b: Acceleration command in the Y-direction for  $150^\circ$  aspect angle,  
 $40^\circ$  off bore-sight angle, and  $3750'$  initial range.

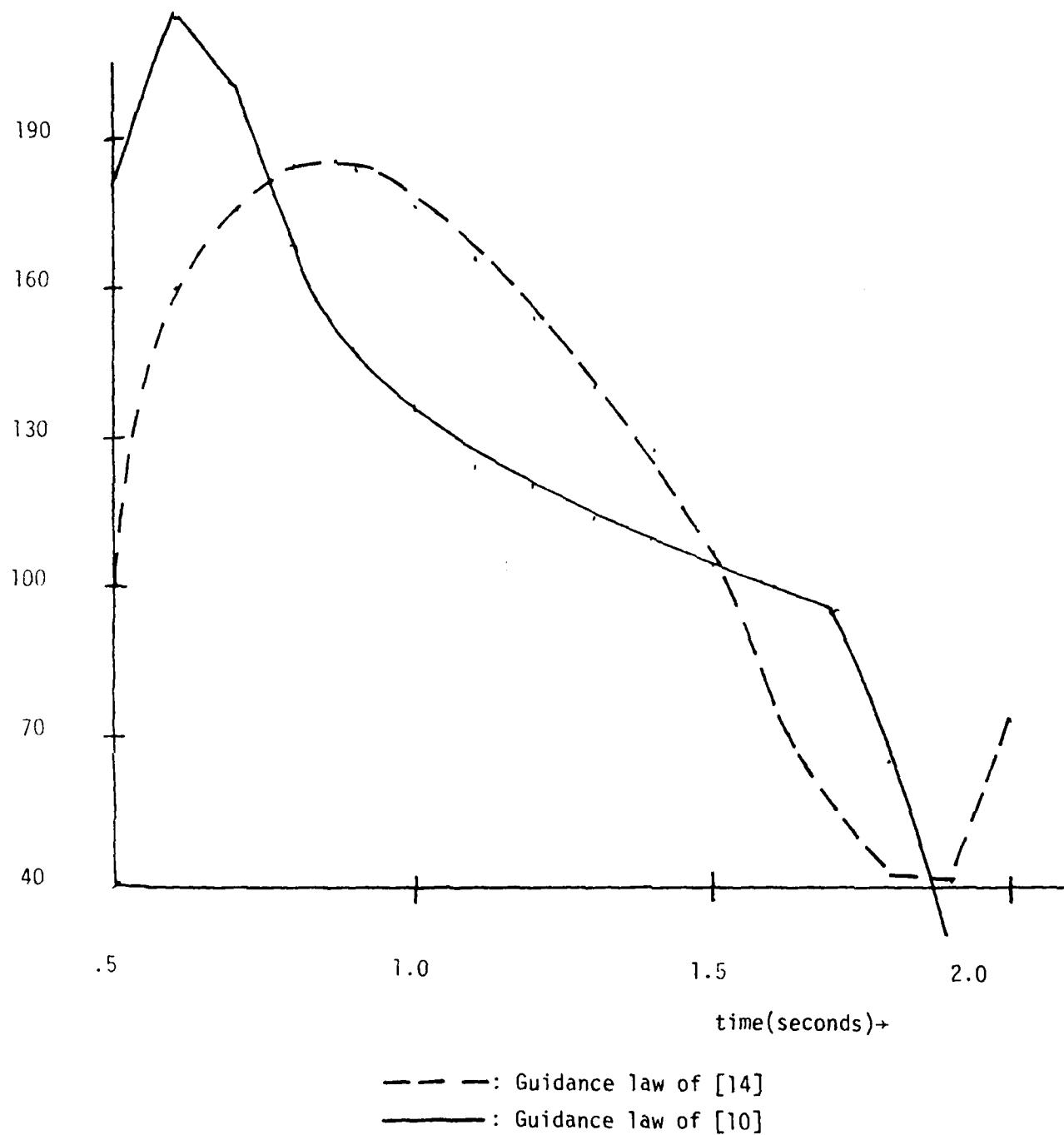


Figure 5c: Acceleration command in the Z-direction for 150° aspect angle, 40° off bore-sight angle, and 3750' initial range.

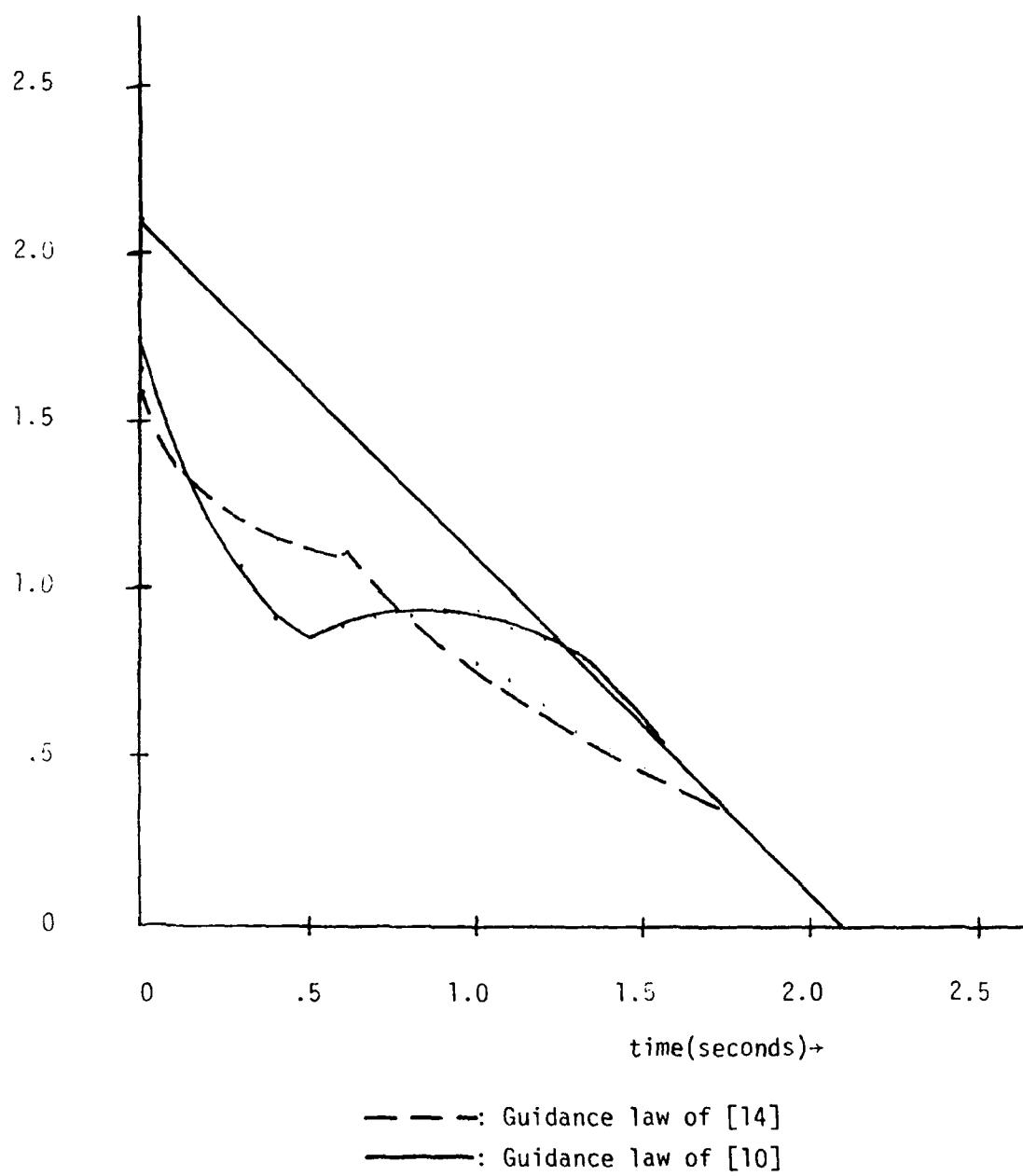
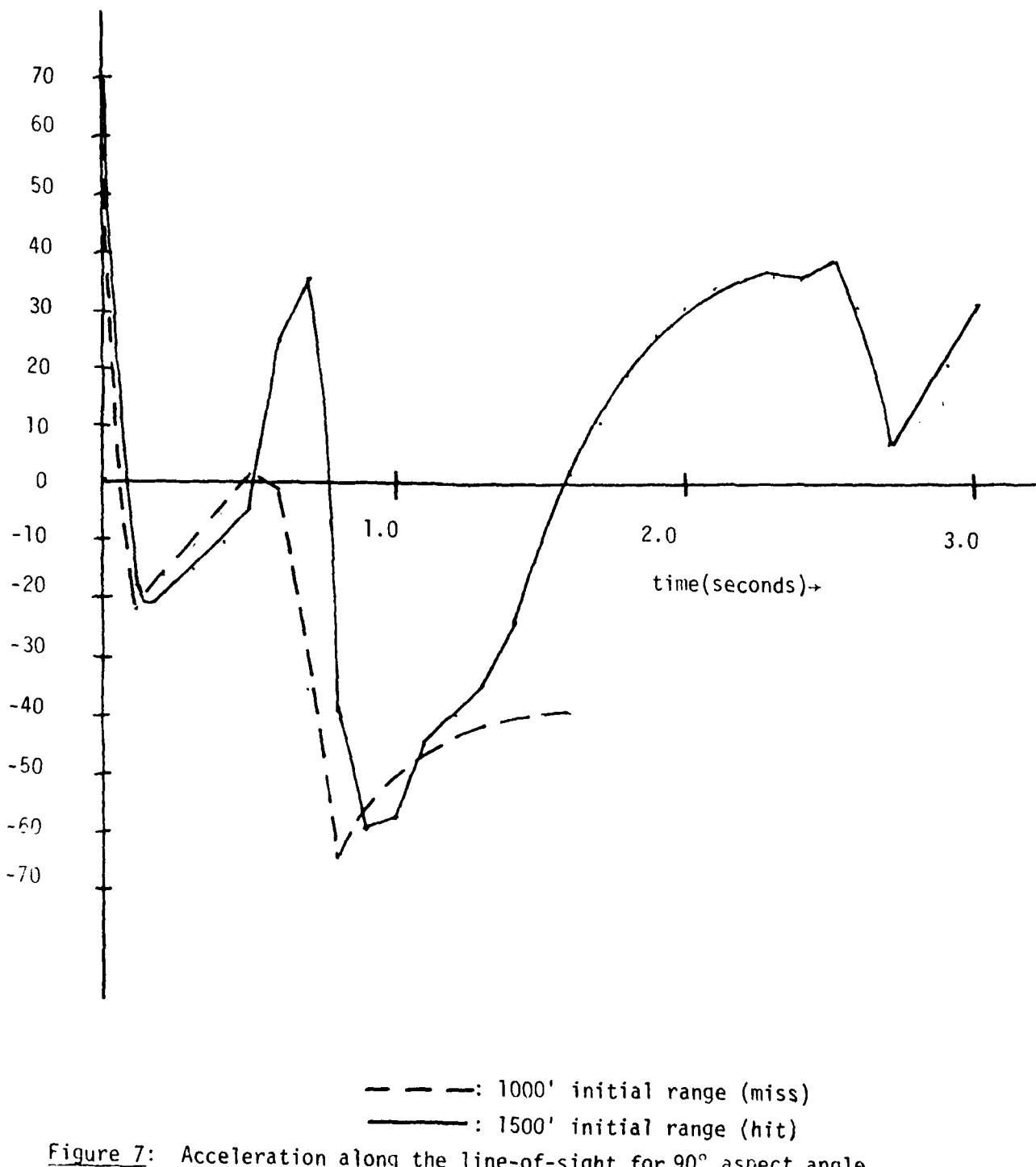


Figure 6: Time-to-go estimate versus true time for  $150^\circ$  aspect angle,  
 $40^\circ$  off bore-sight angle, and  $3750'$  initial range.



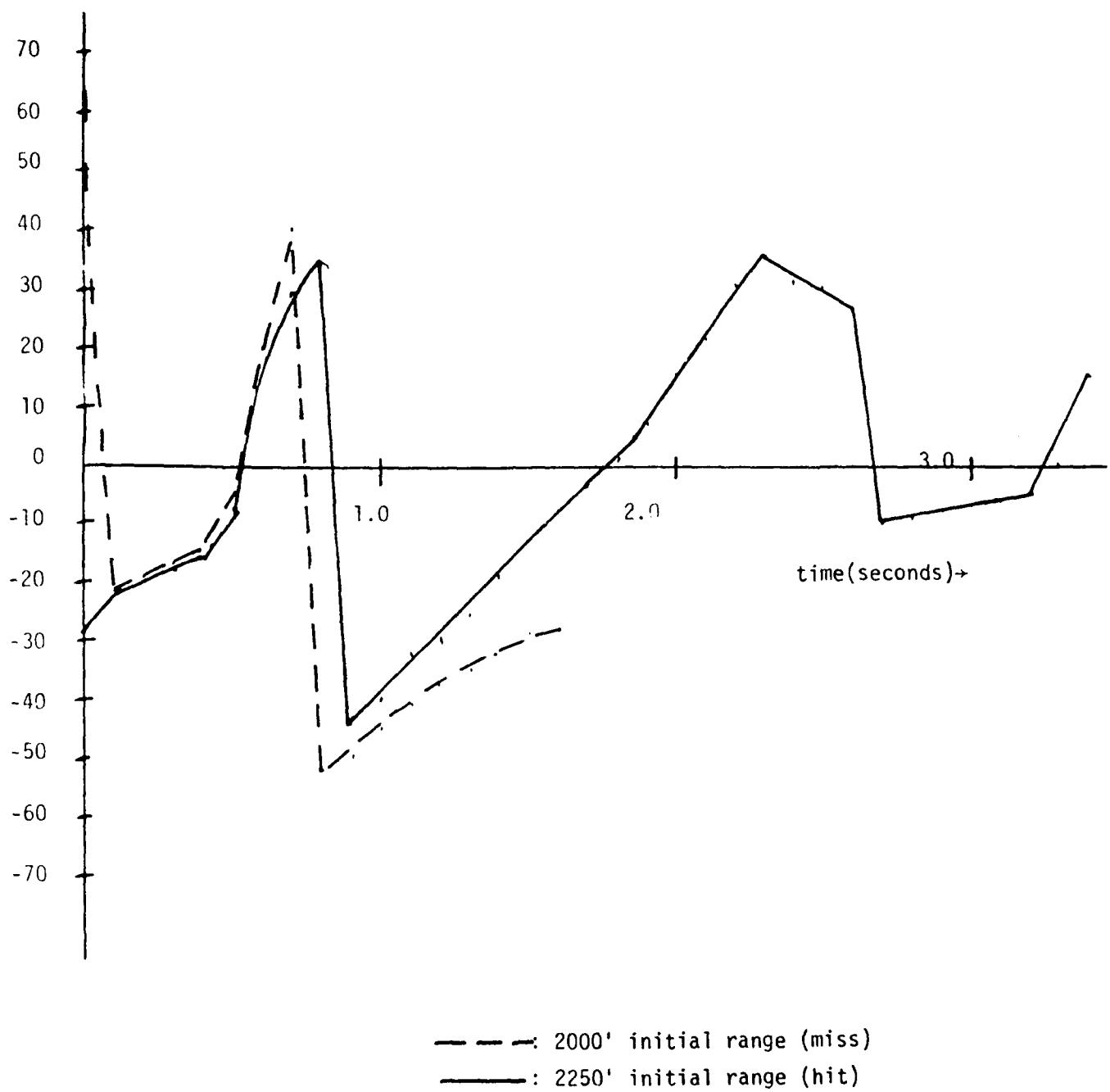


Figure 8: Acceleration along the line-of-sight for 120° aspect angle.

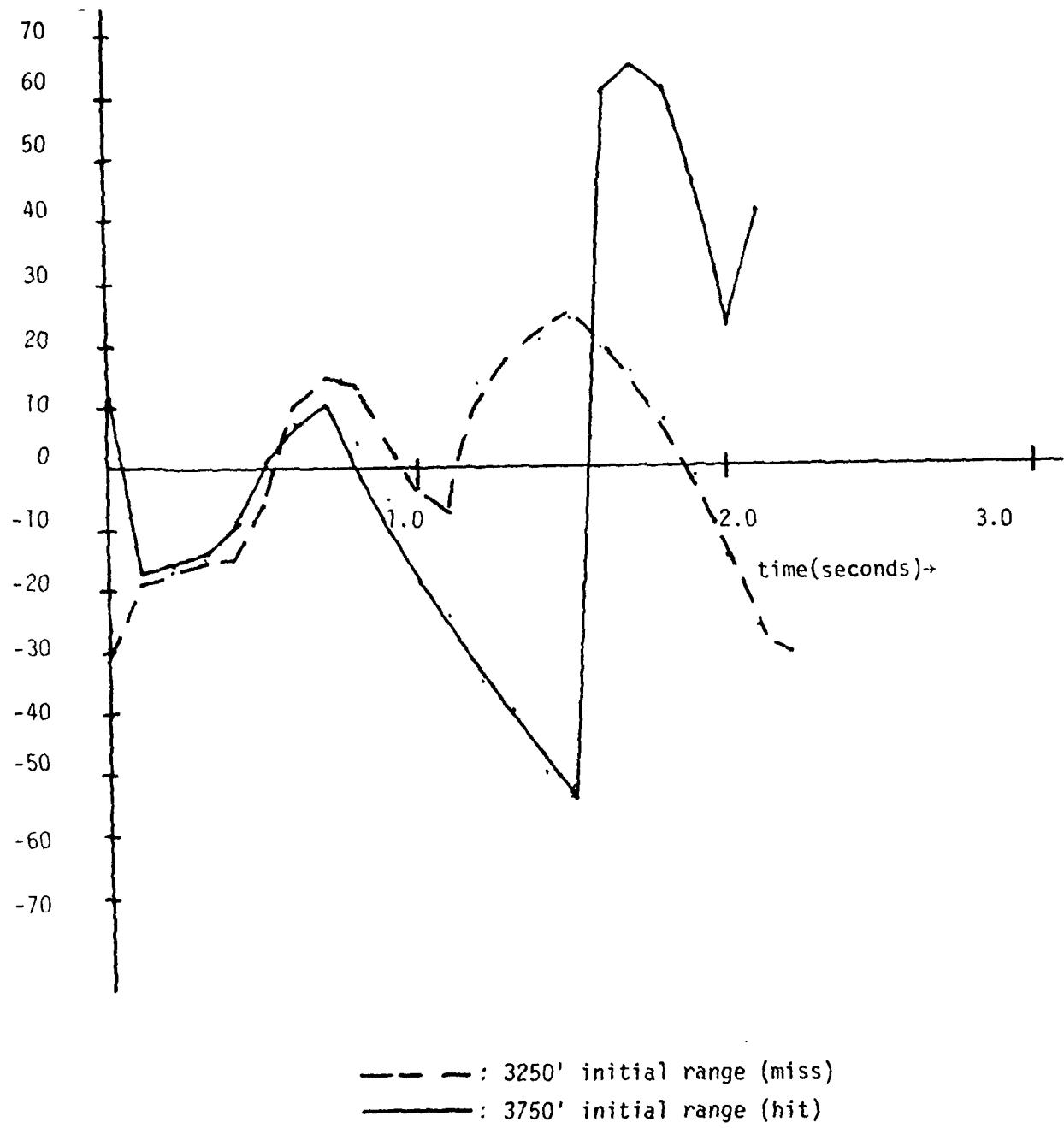


Figure 9: Acceleration along the line-of-sight for  $150^\circ$  aspect angle.

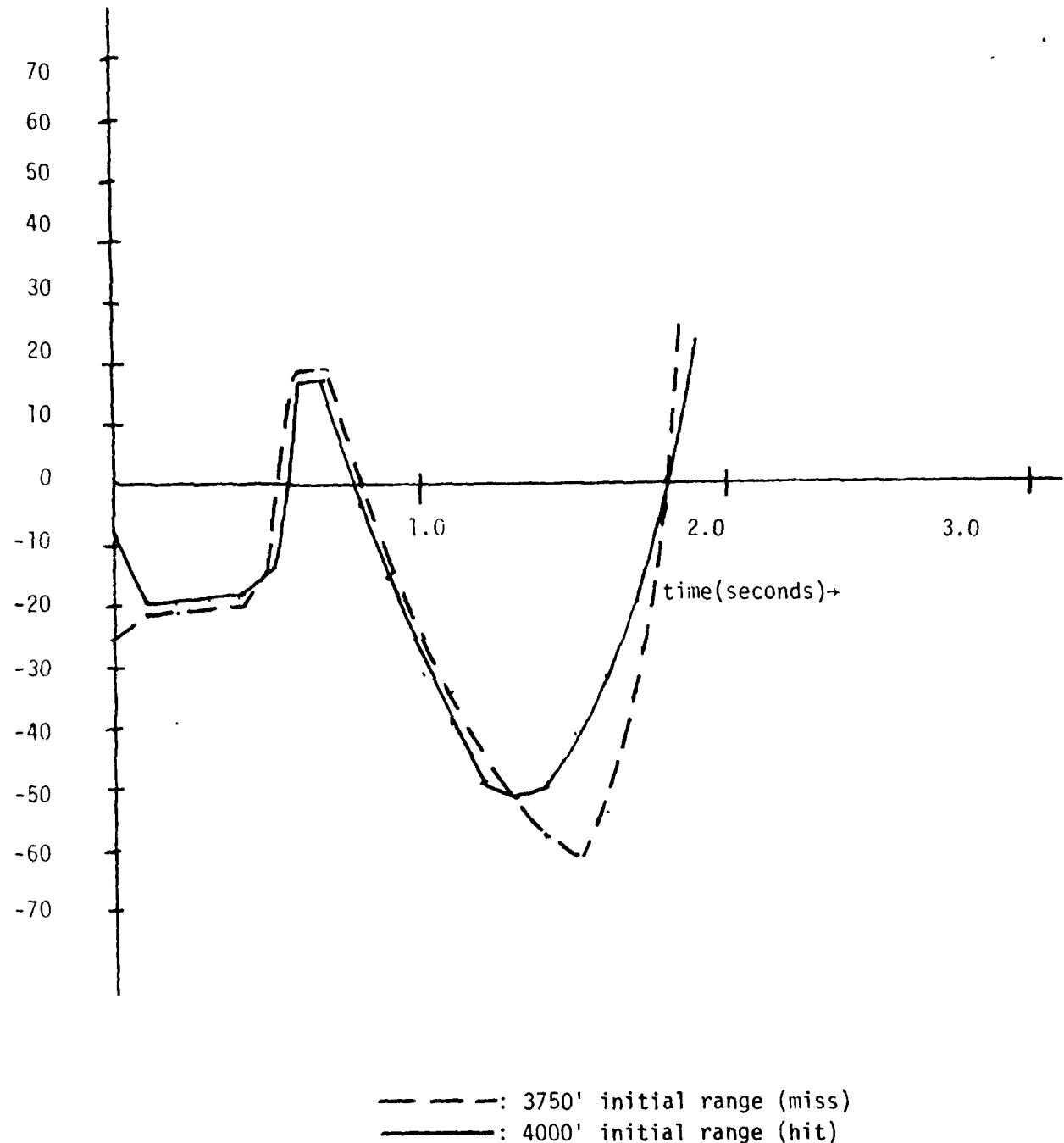


Figure 10: Acceleration along the line-of-sight for 180° aspect angle.

<u>Acceleration Commands</u>		
<u>X-Direction</u>	<u>Y-Direction</u>	<u>Z-Direction</u>
75.00	75.00	75.00
75.00	75.00	75.00
75.00	-13.768	75.00
75.00	2.064	75.00
65.829	1.414	75.00
47.546	1.474	75.00
46.886	2.999	75.00

Table 1: Acceleration commands (sampling period is .1 sec)

This particular case resulted in a miss when constraints were invoked on the missile's control commands. The corresponding time-to-go estimate is shown in Figure 11. Observing Figure 3 and Table 1, one notes that the acceleration commands in the Z-direction are the most critical parameters. Limiting the control in this direction has the most effect on the time-to-go parameter. To compensate for this constraint effect, one may modify the estimator performance measure in (5). This is the subject of the next section.

#### B. Modification of the Time-To-Go Estimator

In order to compensate for non-zero target acceleration or other assumptions (or constraints) on (8), one may modify (5) by making the weight on the minimal time measure time-varying or a function of system parameters.

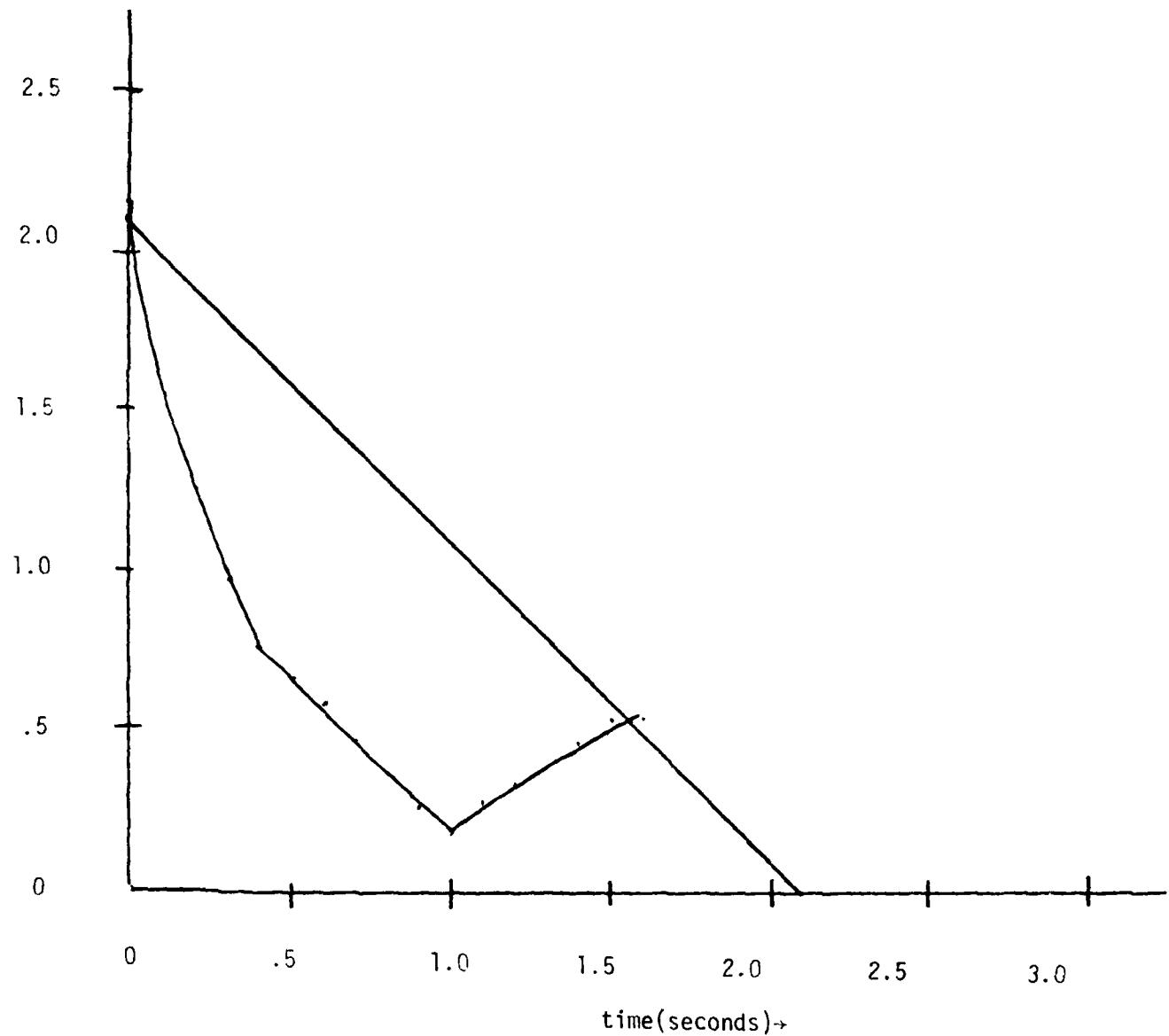


Figure 11: Time-to-go estimate versus true time for constrained control case.

To see the effects of this weighting on the time-to-go estimate, two simulation studies were performed on the case when the aspect angle was  $30^\circ$ , the initial launch range was 2000', and the off bore-sight angle was  $0^\circ$ . The results are shown in Figures 12 and 13. In Figure 12, the time-to-go estimate trajectory for equal weight place on terminal miss distance and minimal time is denoted by a solid line. When the minimal time term is weighted by a 10% increase, the time-to-go trajectory is less conservative initially, signified by a lower dip below the true time line. For more complexed scenarios, this led to more frequent miss distance.

When the minimal time term is weighted by a 10% decrease, the time-to-go trajectory is more conservative initially, signified by a smaller dip below the true time line. In all cases for this particular scenario, the missile hit the target.

To see the effects of initial weighting on the time-to-go estimate, the above three cases were used during the first 0.4 seconds of the launch (until the missile clears its launch platform). Then the initial condition was used in (6) to continue the estimation algorithm. From Figure 13, one observes that this has little effect on the time-to-go estimate for the  $0^\circ$  off bore-sight case.

Hence it was desired to observe the effects of aspect angle and off bore-sight angle on the time-to-go estimate. In particular, the

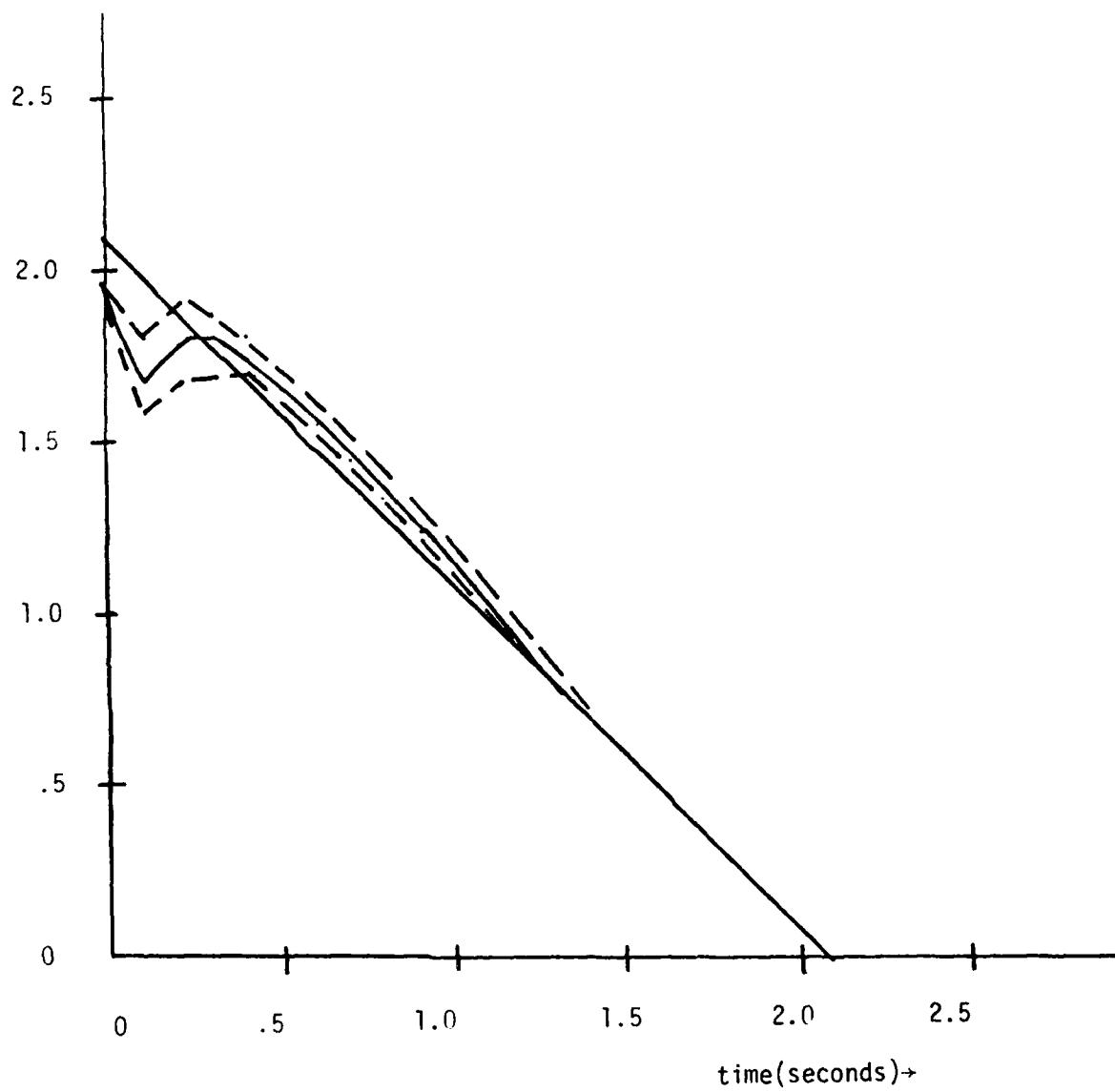


Figure 12: Effects of weighting on the time-to-go estimate for  
30° aspect angle, 0° off bore-sight angle, and 2000'  
initial range.

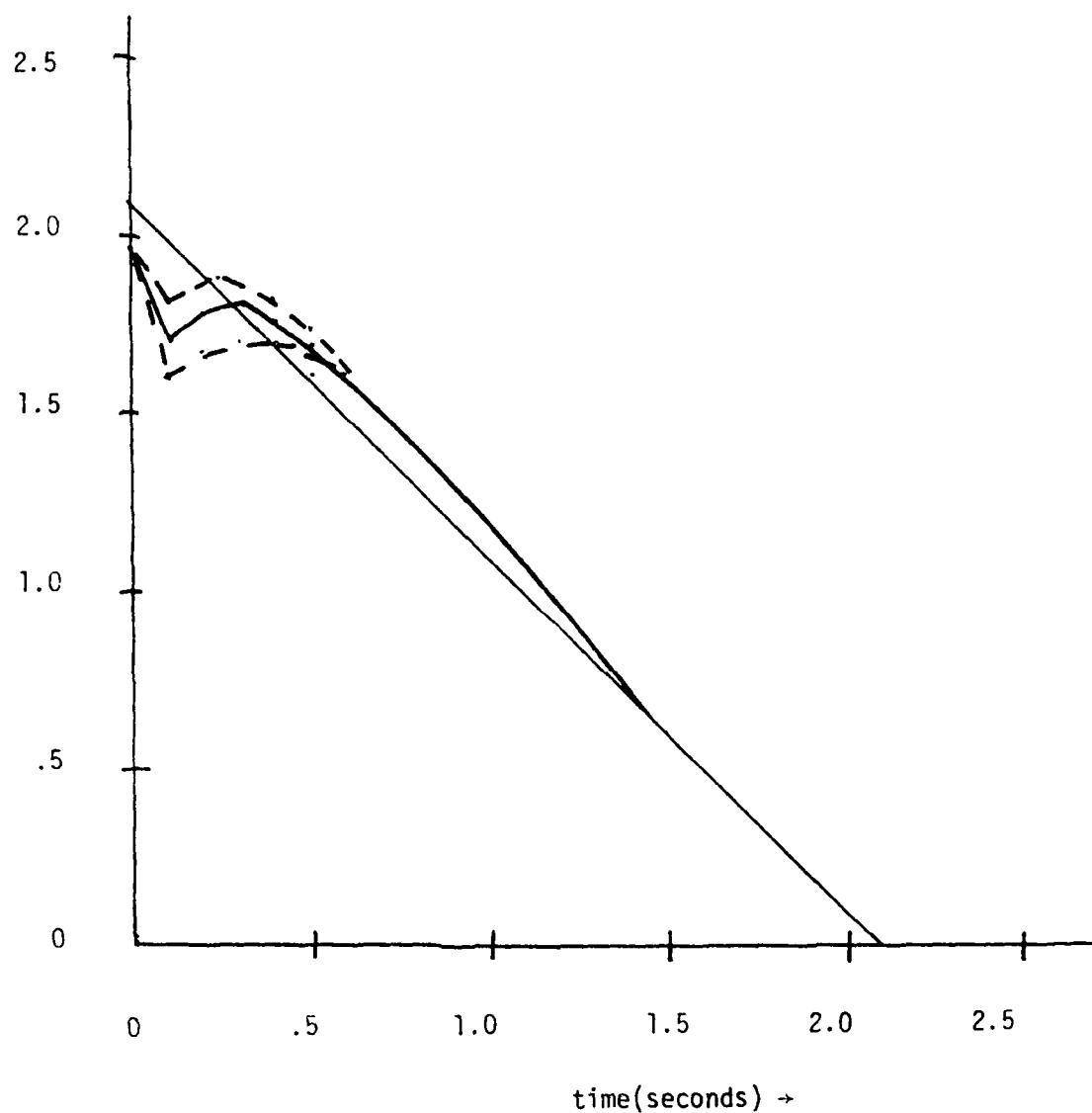


Figure 13: Effects of initial weighting values on the time-to-go estimate for 30° aspect angle, 0° off bore-sight angle, and 2000' initial range.

weighting value  $q$  was chosen to be a functional of the aspect angle or off bore-sight angle, assuming these parameters are known.

After several simulation studies, it was found that a weighting of:

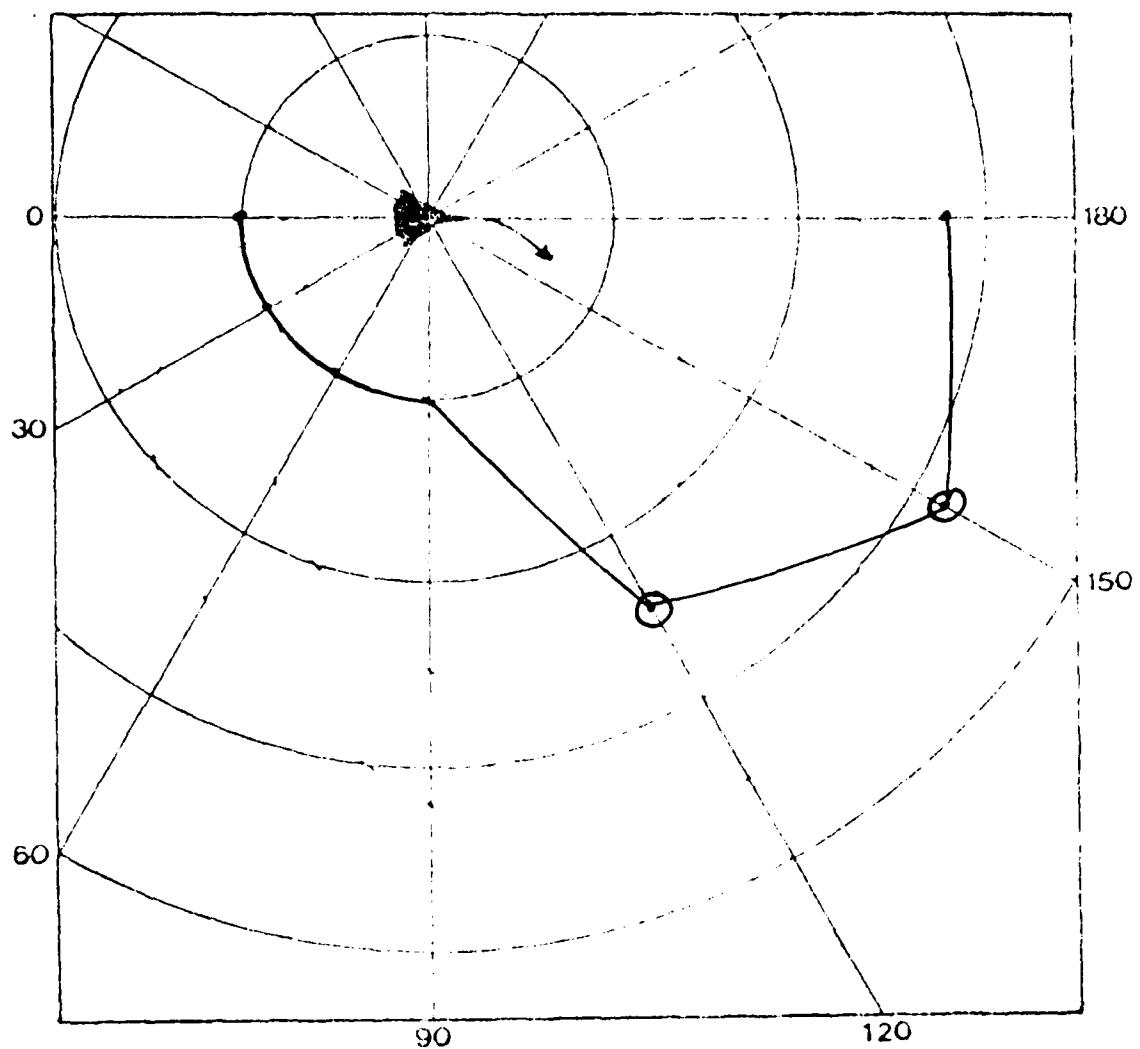
$$q = 0.1 q_0 \cos (\text{AAY}/\text{RADC}) \quad (24)$$

gave improved results, where  $q_0$  is the constant normalized weight chosen in [10], AAY is the aspect angle, and RADC =  $2\pi$ .

This weighting was used in (6) to estimate the time-to-go parameter. A summary of the resulting simulation studies for the  $0^\circ$  off bore-sight angle and the  $40^\circ$  off bore-sight angle is shown in Tables 2 and 3. The inner launch boundaries of these cases are also plotted in Figures 14 and 15.

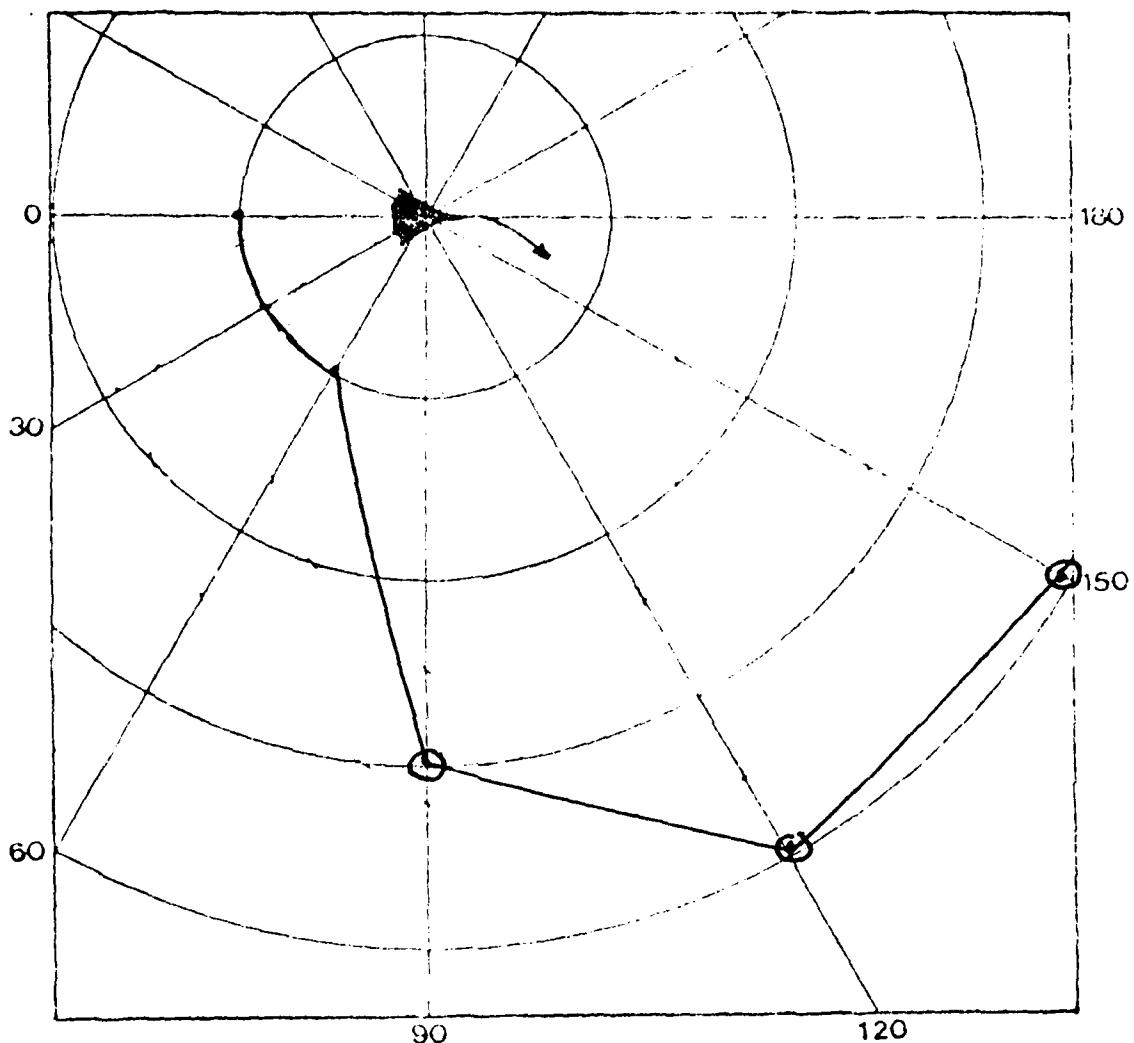
Other more complexed weighting functionals, including the addition of the off bore-sight angle were tried. It was found, however, that these more complexed weights did not improve the time-to-go estimate significantly over that using (24). However, other weighting functionals using other system parameters may further improve this estimate and is the subject of further investigative research.

In summary, the time-to-go estimator described in (6) with modifications in  $q$  has resulted in an improved estimate. One may apply other weightings to improve the estimate even more. However, it is felt that research in the modification of the guidance control laws would be of equal or greater significance.



○: signifies upper bound. Actual inner launch boundary may be smaller (computer simulation stopped before completion of analysis).

Figure 14: Inner launch boundary corresponding to Table 2.



○ : signifies upper bound. Actual inner launch boundary may be smaller (computer simulation stopped before completion of analysis)

Figure 15: Inner launch boundary corresponding to Table 3.

0° off bore-sight angle

aspect angle:	0	30	60	90	120	150	180
range:	1000	1000	1000	1000	2500*	3250*	1875

\* signifies upper bound. Actual inner launch boundary may be smaller.

Table 2: Inner launch boundary using (24)

40° off bore-sight angle

aspect angle:	0	30	60	90	120	150	180
range:	1000	1000	1000	3000*	4000*	4000*	5000*

\* signifies upper bound. Actual inner launch boundary may be smaller.

Table 3: Inner launch boundary using (24)

III. Dynamic Output Feedback for System Regulation

The previous section addressed the issue of modifying the time-to-go estimate due to unrealistic system assumptions in the AAM scenario.

It was found that varying the weighting of (6) resulted in some improvement in the estimate. The reason for this analysis is primarily to improve the guidance laws governed by (14) and (15).

Consider, however, modification of the linear regulator performance measure directly in order to improve the guidance control laws. The subject of this and the following sections is the development of a linear regulation design methodology which can be applied towards the AAM guidance problem. The infinite horizon time case is presented in this section while a discussion of the finite time case is presented in the following section as a basis for future research in optimal guidance and control.

Consider a linear, time-invariant, controllable and observable system represented by:

$$\begin{aligned}\dot{\underline{x}}(t) &= A \underline{x}(t) + B \underline{u}(t) \\ \underline{y}(t) &= C \underline{x}(t)\end{aligned}\quad (25)$$

where there are  $n$  states,  $m$  inputs, and  $r$  outputs. The infinite final time linear regulator problem is to select a control law  $\underline{u}(t)$  such that

$$J = \frac{1}{2} \int_0^{\infty} [\underline{x}^T(t) Q \underline{x}(t) + \underline{u}^T(t) R \underline{u}(t)] dt \quad (26)$$

is minimized, where  $(\cdot)^T$  denotes transpose,  $\underline{x}_0 = 0$ ,  $Q$  is a positive semidefinite matrix, and  $R$  is a positive definite matrix.

If all of the states are accessible, the solution to (26) is well-known and is given by [3]:

$$\begin{aligned}\underline{u}^*(t) &= F^* \underline{x}(t) \\ F^* &= -R^{-1} B^T P^*\end{aligned}\quad (27)$$

where  $P^*$  is the unique symmetric positive definite matrix solution to:

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (28)$$

The corresponding optimal cost functional is:

$$J = \frac{1}{2} \underline{x}^T(0) P^* \underline{x}(0) \quad (29)$$

If state feedback is not feasible, consider an output dynamic compensator represented by:

$$\dot{\underline{x}}_c(t) = A_c \underline{x}_c(t) + B_c \underline{y}(t) \quad (30)$$

where  $n_c$  is the order of the compensator. The output feedback control law may then be given as:

$$\underline{u}(t) = C_c \underline{x}_c(t) + D_c \underline{y}(t) \quad (31)$$

Some of the elements of (30) and (31) may be constrained to be certain values given a priori. One may augment (25) and (30), using (31), to obtain:

$$\dot{\underline{x}}_a(t) = (A_a + B_a K C_a) \underline{x}_a(t) \quad (32)$$

where

$$A_a = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \quad B_a = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \quad C_a = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$$

$$K = \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} \quad \underline{x}_a = \begin{bmatrix} \underline{x} \\ \underline{x}_c \end{bmatrix} \quad (33)$$

The output linear regulator problem then is to select  $(A_c, B_c, C_c, D_c)$  as well as  $n_c$  to minimize (26). In order to do this, consider defining an error vector

$$\underline{e}(t) = \underline{x}(t) - \underline{x}^*(t) \quad (34)$$

where  $\underline{x}^*(t)$  is the optimal state trajectory obtained from (27) using state feedback and  $\underline{x}(t)$  is the state trajectory using (30). The error vector follows a time trajectory described by:

$$\dot{\underline{e}}(t) = (A + BF^*) \underline{e}(t) + B[(D_c C - F^*) \underline{x}(t) + C_c \underline{x}_c(t)] \quad (35)$$

Define a excitation vector:

$$\underline{g}(t) = (D_c C - F^*) \underline{x}(t) + C_c \underline{x}_c(t) \quad (36)$$

or equivalently

$$\underline{q}_a(t) = (KC_a - F_a) \underline{x}_a(t) \quad (37)$$

where

$$\underline{q}_a = \begin{bmatrix} q \\ 0 \end{bmatrix} \quad F_a = \begin{bmatrix} F^* & 0 \\ 0 & 0 \end{bmatrix} \quad (38)$$

Then

$$\underline{e}(t) = (A + BF^*) \underline{e}(t) + B\underline{q}(t) \quad (39)$$

Note that if  $\underline{q}(t)$  is bounded,  $e(t)$  is bounded and stable.

Consider, then, minimizing

$$\hat{J} = \frac{1}{2} \int_0^\infty \underline{q}^T(t) R_q \underline{q}(t) dt \quad (40)$$

where  $R_q$  is a symmetric positive definite matrix. Using (33), one finds:

$$\hat{J} = \frac{1}{2} \int_0^\infty \underline{x}_a^T(t) [KC_a - F_a]^T R_a [KC_a - F_a] \underline{x}_a(t) dt \quad (41)$$

where

$$R_a = \begin{bmatrix} R_q & 0 \\ 0 & 0 \end{bmatrix} \quad (42)$$

If a minimum for  $\hat{J}$  exists, then the error excitation is minimum over the total time interval. Hence, a minimum deviation of the state trajectories from the optimal state trajectories will be achieved. The output regulation problem therefore is to select  $K$  to minimize (41) subject to (32).

Since (41) is in standard form, the optimal  $\hat{J}$  is given by:

$$J(K^*) = \frac{1}{2} \underline{x}_a^T(0) P_a^* \underline{x}_a(0) \quad (43)$$

where  $P_a^*$  is the solution to the Lyapunov Matrix Equation:

$$(A_a + B_a K^* C_a) P_a + P_a (A_a + B_a K^* C_a) + [K^* C_a - F_a]^T R_a [K^* C_a - F_a] = 0 \quad (44)$$

Note that (44) requires knowledge of  $K^*$ , the optimal compensator parameters.

One can solve (43) and (44) numerically by augmenting (43) as:

$$\begin{aligned}\hat{J}(K, P_a, r) = & \frac{1}{2} \underline{x}_a^T(0) P_a \underline{x}_a(0) \\ & + \frac{1}{2} \text{tr} \{ r[(A_a + B_a K C_a) P_a + P_a (A_a + B_a K C_a)] \\ & + (K C_a - F_a)^T R_a (K C_a - F_a) \} \end{aligned}\quad (45)$$

where  $r$  is the Lagrange Multiplier and  $\text{tr}(\cdot)$  denotes the trace of a matrix.

Minimization of (45) requires construction of the following gradient matrices:

$$\begin{aligned}\frac{\partial \hat{J}}{\partial P_a} = & (A_a + B_a K C_a) r + r(A_a + B_a K C_a)^T \\ & + \underline{x}_a(0) \underline{x}_a^T(0) = 0 \\ \frac{\partial \hat{J}}{\partial r} = & (A_a + B_a K C_a)^T P_a + P_a (A_a + B_a K C_a) \\ & + (K C_a - F_a)^T R_a (K C_a - F_a) = 0 \\ \frac{\partial \hat{J}}{\partial K} = & [B_a^T P_a + R_a (K C_a - F_a)] r C_a^T = 0 \end{aligned}\quad (46)$$

This completes the necessary equations of optimality for the infinite final time cost functional described by (26).

#### IV. Application of the Error Excitation Approach to the Finite Terminal Time Case

When  $t_f$  is finite, the optimal state feedback control law is given by (10) and (11) resulting in a two-point boundary value problem. Using the error

vector approach with dynamic compensation; the optimal control law is given by (30) and (31), found by minimizing (40). The minimum error excitation value is:

$$\hat{J}(k^*) = \frac{1}{2} \underline{x}_a^T(0) P_a^*(t_0) \underline{x}_a(0) \quad (47)$$

where  $P_a^*(t)$  satisfies:

$$\begin{aligned} \dot{P}_a(t) + (A_a + B_a K^*(t) C_a) P_a(t) + P_a(t) (A_a + B_a K^*(t) C_a) \\ + (K^*(t) C_a - F_a(t))^T R_a (K^*(t) C_a - F_a(t)) = 0 \end{aligned} \quad (48)$$

with  $F_a(t) = \begin{bmatrix} F^*(t) & 0 \\ 0 & 0 \end{bmatrix}$

and

$$\begin{aligned} \underline{x}_a^*(t_0) &= \underline{x}_a(t_0) \\ P^*(t_f) &= S_f \end{aligned} \quad (49)$$

Note that  $K^*$  must be found by solving a two-point boundary value problem.

The terminal time linear regulator problem using state or dynamic output feedback resulted in a two-point boundary value problem which usually requires numerical techniques for a solution. Consider, however, the air-to-air missile scenario in which the intermediate states need not be regulated and the weight on the control effort is zero ( $r=0$ ). In this case, the optimal state trajectories take on the described by:

$$\dot{\underline{x}}^*(t) = (A + BF^*) \underline{x}(t) \quad (50)$$

that is:

$$\underline{x}^*(t) = e^{\bar{A}(t-t_0)} \underline{x}(t_0) \quad (51)$$

where

$$\begin{aligned} \bar{A} &= A + BF \\ &= \begin{bmatrix} 0 & I \\ -\frac{3}{t_{g_0}^2} - I & -\frac{3}{t_{g_0}} - I \end{bmatrix} \end{aligned} \quad (52)$$

In this case, the problem of (45) and the associated gradient matrix equations of (46) have a reduction in numerical computation which makes the two point boundary value problem more feasible to implement.

The algorithm described by (45) and (46) has been programmed and applied to a multitude of problems. The application of this design methodology to the air-to-air problem is a fruitful area of future research. Extensions of (45) to the time-varying and stochastic case also have direct application to the AAM guidance problem.

#### V. Summary and Recommendations

The purpose of this research effort was to investigate the effects of target and missile parameters on the time-to-go estimation algorithm developed by the author in [10]. It was found that the addition of acceleration command constraints had the most effect on the Z-direction control. In all cases tested, the constraints drastically increased the terminal miss distance. In the test case presented in this report for a simple off bore-sight angle, the additional constraints caused the missile's

guidance strategy to result in a miss (under unlimited control, the missile came within 2' of the target).

To compensate for this constraint limitation as well as other assumptions on the time-to-go estimation development, it was necessary to incorporate more information into the weighting functional of the estimator. It was found that the addition of aspect or off bore-sight angle resulted in an improvement of the time-to-go estimate.

Finally it is noted that the ultimate objective of guidance and control is to develop optimal control laws under various scenarios. Towards this objective, it is felt that a modification of the linear regulator formulation is necessary to improve the guidance laws of the AAM problem.

The design methodology presented here represents regulation under output or structured state feedback compensation. The algorithm can be applied to the stochastic or time-varying case. Stability of the regulator is also insured using this approach, which may not be the case in many algorithms. It is felt that the application of this methodology to the AAM problem would yield significant results.

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